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Numerical enclosures of the optimal cost of the Kantorovitch's mass transportation problem

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Abstract The problem of optimal transportation was formalized by the French mathematician Gaspard Monge in 1781. Since Kantorovitch, this (generalized) problem is formulated with measure theory. Based on Interval Arithmetic, we propose a guaranteed discretization of the Kantorovitch's mass transportation problem. Our discretization is spatial: supports of the two mass densities are partitioned into finite families. The problem is relaxed to a finite dimensional linear programming problem whose optimum is a lower bound to the optimum of the initial one. Based on Kantorovitch duality and Interval Arithmetic, a method to obtain an upper bound to the optimum is also provided. Preliminary results show that good approximations are obtained.

Keywords Optimal transportation · Interval arithmetic · Continuous programming · Optimization

1 Introduction

Optimal Transportation is a mathematical research topic which started with Monge theory "des remblais et d blais" in 1781. In the 1940s, Kantorovitch [11] gave the modern formulation of this problem. This problem is to minimize the transport cost between two mass densities μ and ν. Without loss of generality, the total mass to

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